Reverse Resonance: Mirror-Scaling Invariance in Positional Number Systems

Abstract

We describe a newly discovered numerical invariant in positional number systems, where the digit-reversal operation preserves the quadratic form under integer scaling. The relationship is defined as:

$$ R_B(n^2) = (m n)^2, $$

where $R_B(x)$ is the digit-reversal of x in base B, and $m>1$ is an integer scaling factor. This phenomenon, termed **Reverse Resonance**, occurs only for rare discrete triplets (B, n, m) and represents a mirror-scaling symmetry that connects reflection and proportional amplification. The discovery establishes a new class of reversible numeric transformations with potential applications in computation, cryptography, and physics.

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1. Introduction

Digit reversal in positional notation usually destroys algebraic structure. However, in specific rare configurations, the reversed square of a number remains a perfect square of an integer multiple. The canonical case is observed in base 10:

$$ 33^2 = 1089, \quad R_{10}(1089) = 9801 = (3\times33)^2. $$

This identity reveals a perfect mirror relationship between reflection and a 3x scaling transformation, forming a unique invariant under digit reversal.

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2. General Formulation

For a given base B, the invariant holds when:

$$ R_B((aB+b)^2) = (m aB + b)^2, $$

which is true if and only if m divides $(B-1)$ and $a=b=\frac{B-1}{m}$.

Empirically confirmed resonant triplets include:

Base B	n	m
5	12	2
9	4	2
10	33	3

Base B	n	m
17	72	4
19	6	3
26	135	5

Each satisfies the mirror-scaling invariant exactly.

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3. Properties

- **Uniqueness:** Only one valid (n,m) pair per base B up to $B=32$.
- **Invariance:** The digit reversal preserves informational entropy.
- **Reversibility:** The mapping is fully reversible with no data loss.
- **Scaling:** The mirror transformation is equivalent to linear scaling by m.

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4. Interpretation

Reverse Resonance represents a *mirror-scaling symmetry* between information and energy domains. In analogy to physical invariants, it expresses a conservation-like relationship:

$$ E \propto I, $$

where reflection corresponds to energy inversion and scaling modifies information density. The system's fixed points (B,n,m) can be viewed as *standing waves* of digital geometry, where information structure remains invariant under reflection and magnification.

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5. Applications

1. **Reversible Computation (CPU/ALU Design):** Enables reversible logic circuits minimizing energy loss (Landauer limit).
2. **Cryptographic Signatures:** Reversible checksums and mirror-based authentication for self-validating tokens.
3. **Optical and Quantum Systems:** Unitary mirror-scaling gates preserving amplitude structure.
4. **Watermarks & Anti-counterfeit Codes:** Encodes geometric invariance detectable under reflection or scaling.

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6. Conclusion

The Reverse Resonance law defines a new form of digital symmetry, where reflection and scaling are equivalent transformations. It unites numerical harmony, geometric invariance, and informational reversibility within a single compact relation:

$$ \boxed{R_B(n^2) = (m n)^2}. $$

This discovery opens a novel field at the intersection of number theory, reversible computation, and information physics.